Editor's Note: Due to a production error, the title of one book was accidentally omitted from the following review of two books, which appeared in the May 1995 issue. In order that the content of the review can be fully understood and appreciated, we reprint the review here and apologize to readers who may have been confused by the error.

Chaos and Nonlinear Dynamics (An Introduction for Scientists and Engineers).

By R. C. Hilborn, Oxford University Press, New York, 1994, 625 pp., $55.00 (hard cover), $22.50 (paperback).

Nonlinear Dynamics and Chaos (with Applications to Physics, Biology, Chemistry, and Engineering).

By S. H. Strogatz, Addison-Wesley, Reading, MA, 1994, 498 pp., $51.95 (hard cover).

The number of serious books in modern aspects of nonlinear science and chaos published since the early 80s is nothing short of staggering. A short list might include Guckenheimer and Holmes, Lichtenberg and Lieberman, Schuster, Thompson and Stewart, Moon, and Wiggins. None of these books can be regarded as complete, and many do not qualify as easy. G&H is heavily tilted toward ODEs, L&L toward Hamiltonian systems, and Schuster to maps. The case could be made that few books, if any, are actually suitable for undergraduate education (one exception might be Baker and Gollub, but the coverage is limited). The two books reviewed here are intended, according to their authors, to fill this void. Strogatz's is directed at upper-level undergraduate or lower-level graduate mathematics students, whereas Hilborn's is directed at physics undergraduates. The very fact that both have subtitles including the word engineering indicates that their authors have significantly larger audiences in mind.

Parallel book reviews make comparisons almost inevitable, and two reviewers just augment the possibilities. A few general points are easy to agree upon: Strogatz's is more focused; Hilborn's, by encompassing more territory, is an easier target for criticism. The titles are also uncharacteristically descriptive; in Strogatz *nonlinear* comes first, in Hilborn it is *chaos*. Both books try to be pedagogical. Strogatz's might be easier to teach from, but Hilborn presents a more complete picture of the current state of research. Strogatz is a mathematician, and his book is crisp and clear; Hilborn is an experimentalist and his book conveys a sense of science in progress ("what you are reading here is not the last word"). Strogatz's development is tidy to the point that everything looks to be completely defined. Both books are excellent, albeit quite different. The following comments are offered in the spirit of anticipating second editions.

Hilborn's book is substantial (654 pp.). It has an amazingly complete list of references—upward of 450 with beginning page and end pages, many with comments. This is a considerable resource. One has to search far and wide to find unnamed classics (such as Wisdom's articles in *Science*). The structure is as follows: Part I, *Phenomenology of Chaos*, uses electrical circuits, population growth, and the Lorenz model as paradigms; Part II, *Toward a Theory of Nonlinear Dynamics and Chaos*, presents an overview of both dissipative and Hamiltonian chaos; Part III, *Measures of Chaos*, is self-explanatory; and the concluding Part IV, *Special Topics*, examines spatial patterns and quantum chaos. Some background in fluid mechanics is provided in this part. The book goes slowly—the preface alone runs six pages (Strogatz's is two pages). It has also complete computer programs and a hefty number of problems, some trivial and others pretty hard.

Hilborn's narrative is pleasant but sporadically long-winded; it is as if the author fears being misunderstood and tries to covers all bases, often explaining and presenting the same information in different ways. For example, he plots both voltage and current for his diode circuit, where either one would do; he plots x, y, and z independently as well as a state-space plot for the Lorenz system. This spills over to the exercises as well; it is as if some of them are simply designed to eliminate every possible reader's doubt.

The style is not entirely uniform. For example, only Chapter 8 starts with a quote. There is also a proliferation of font sizes which is distracting, and the figures are not uniform—some are very small and others quite large. A similar criticism can be levied on the coverage. For example, the Brusselator equations are not derived (although a charming anecdote for the origin of the name is described), but Van der Pol oscillator equations are derived in four pages. But then the Brusselator is exhaustively analyzed, while the Van der Pol system is not. Later on, the book assumes that the student knows about Taylor series, ODE's, etc., but presents eigenanalysis in exhausting detail.

In spite of these observations, there is much good that can be said about Hilborn's book, most of which has to do with completeness. Completeness and coherence, however, are hard to reconcile. By design, books are one-dimensional; it is permissible to refer to past material and only sparingly to discuss what lies ahead. However, as anyone who has tried to write a book can testify, putting an entire field together and conforming to this simple precept are not easy. Here we run into this problem. The text often refers back and forth; fractals appear on p. 35, but nothing is done with them for over 300 pages. Similarly, Hilborn refers to dimensions early on but does not treat them until much later. The mathematics is sometimes rigorous and sometimes not. For example, he does not seem to distinguish between a stationary point in a flow and a fixed point in a map. The first definition of the Lyapunov exponent, on the other hand, is overly precise and not the intuitive one (p. 84). Later (p. 176), he mentions some cautions about the Lyapunov exponent, but neglects recent work dealing with limitations due to finite time. Notwithstanding these minor shortcomings, his treatment of crises is quite pretty and the horseshoe treatment is also very nice. In addition he gets bonus points for mentioning the importance of error analysis in dimensional estimates. The above comments should not distract us from the central point that Hilborn's book is remarkably complete in its coverage, can serve as an excellent introduction to a number of topics, and can be used to get quick access to the state of the art.

Strogatz's book is less ambitious in its coverage and is by comparison less physical, but is more rigorous mathematically. The book is better produced and it has some color plates, though not much new comes out of them. It has a remarkably good subject index (upward of 22 pages, Hilborn's has a bit more than 5). It has fewer references than Hilborn's (211 vs. 450 for Hilborn's). On the other hand, it has an amazingly large number of problems—581, with one chapter having 89—and about a quarter have solutions.

Strogatz's *Nonlinear Dynamics and Chaos* starts from the very beginning of dynamical systems, assuming a knowledge of little more than simple differentiation, and systematically but elegantly covers virtually the entirety of modern mathematical dynamics theory. It would be difficult to design a more concise coverage of the subject.

Initial chapters lay the foundations by explaining fundamentals of one-dimensional flow dynamics. The treatment is at once intuitive and rigorous, with copious, practical examples from the biological chemical and physical sciences. To illustrate uniqueness in simple flows, Strogatz brings up the "leaky bucket" problem. To explore fundamental bifurcations, Strogatz discusses transitions in the laser and the bead-on-a-rotating-hoop. Early on, Strogatz even touches on catastrophe theory using an example of budworm population dynamics. To complete his treatment of one-dimensional dynamics, Strogatz studies flows on a circle. Here, he introduces his forte, synchronization of coupled oscillators. Again, he illustrates his points with two intriguing examples. First, he introduces his own model for synchronization of Asian fireflies, which by itself is delightful. He follows this up with a second example, this time from condensed matter physics: the superconducting Josephson junction. Far from being daunting, Strogatz's treatment of even this potentially thorny subject is concise, simple, self-contained and, we think, fully comprehensible to any serious reader.

Having completed the preliminaries, Strogatz then turns to the heart of the book, where he investigates the dynamics of two-dimensional flows. Here, he really turns it on. He introduces the reader to many different types of flows in phase space and just when the going might get tough, he discusses the Romeo and Juliet problem. $R(t)$ defines Romeo's love/hate for Juliet, and $J(t)$ defines Juliet's love/hate for Romeo. Set up some simple equations, and the action begins. For some parameter values, the two fickle lovers enter an endless cycle of love and hate; for others the lovers can spiral in and out of love or interact explosively. While some readers may be put off by the light-hearted approach, most students who have sat through dry, unenlightening coverage of an important topic will be able to identify at once this inspired treatment.

The fun examples do not stop there: Strogatz studies the tracking by a dog of a duck in a pond; he recounts the history of the oscillatory Belousov-Zhabotinsky chemical reaction; he analyzes a model for genetic switching. Yet each of these examples is no more than a touch point. As readers travel from one to the next, they pick up new, fundamental, dynamical tools from bifurcation diagrams to two-timing to null clines to Lyapunov functions. All treated in a uniformly solid and concise manner.

Finally, Strogatz introduces chaos. Strogatz's treatment of chaos is self-contained, and the advanced or ambitious reader could start immediately in on the chaos chapters. Here, he studies the classic examples of dissipative chaos: the Lorenz, the Logistic and the Hénon equations. He first studies the Lorenz equation, and characteristically he introduces a simple water-wheel model which has been used to mimic the chaotic motion of Lorenz' equations. There is an entire chapter on the Lorenz equation, but remarkably the equations themselves are simply written without much discussion or derivation—not an entirely bad thing since these equations are rederived in many books. At the same time, however, in this very chapter there is lengthy derivation of an *ad-hoc* continuity equation and volume contraction formulae. Again, characteristically he skims the cream of the subject by recounting Cuomo's demonstration that two chaotic circuits can be synchronized and used to transmit speech. He explores quasiperiodicity only briefly, in an exercise, and then moves on to study universality in the logistic map. He stresses the important points in a clear-cut fashion, excising definitive plots from the literature, for example showing that an extreme magnification of the logistic bifurcation plot is virtually identical to its parent and that the sine map and the logistic map produce qualitatively very similar bifurcation plots. Thus the quantitative renormalization treatment—including mention of the seminal results of Libchaber and, of course, Feigenbaum—follows naturally.

Strogatz concludes with one brief chapter each on fractals, and on strange attractors. Many people would like to see more on these topics as well as spatio-temporal and Hamiltonian chaos, multifractals, pattern formation, and so on. As a text, this book already covers much more than a full semester of material. It is likely, however, that many novice readers will find the treatment of some of the most popularized topics in the field to be limited. For this reason, many readers and educators may want to complement Strogatz's enjoyable, mathematically inclined text with a broader and more phenomenogical treatment such as that provided by Hilborn.

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